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ECONOMIES WITH COALITIONAL STRUCTURES AND CORE-LIKE EQUILIBRIUM CONCEPTS

Robert P. Gilles

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ECONOMIES WITH COALITIONAL STRUCTURES

AND CORE-LIKE EQUILIBRIUM CONCEPTS[+]

by

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Abstract

The purpose of this paper is to give a new and more realistic foundation to the modelling of large exchange economies, i.e. economies with perfect competition. After the fundamental work by Aumann (1964,1966) several authors tried to solve some shortcomings of the model, especially the lack of a consistent economic interpretation. However the limit-approach of Hildenbrand (1974), the recent work by Armstrong and Richter (1984,1986) on economies with coalitions only, and the f-core approach by Hammond, Kaneko and Wooders (1987) do not give a satisfactory answer to this problem.

In this paper we introduce a new primitive concept to the model: the coalitional structure of agents, in which the agent has as well individualistic as social characteristics. Modelled with non-classical measure theory this approach seems to have a consistent economic interpretation.

Moreover, our modelling leads to a differentiated coalition concept, and thus to several possibilities for refinement of the core. In this way we introduce the semi- and contract-core as refinements of the original (Aumann) core. We conclude our investigations by stating under which conditions these new core-like equilibrium concepts are equal to the usual core, and under which conditions the semi-core is equal to the set of Walrasian equilibrium allocations. (An extension of the famous Aumann equivalence theorem.)

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<u>CONTENTS</u>

1. INTRODUCTION

The main purpose of this paper is to present a new framework for the modelling of economies with perfect competition. After the seminal work by Aumann (1964, 1966), in which he presented a measure theoretic model to describe perfect competitive economies, many authors tried to solve the problems which are raised by the use of measure theory. First we will try to sketch these problems.

In these models it is assumed that an economic agent is negligible in the process that leads to the Walras equilibrium, but is otherwise effective in the pursuit of his/her interests when he/she acts cooperatively. Within this setting the modeller builds the model around a certain primitive concept which is described with the use of measure theoretic tools. Finally the model is completed by the construction of the Walrasian equilibrium as the solution concept for non-cooperative behavior, and some core-like equilibrium concept for cooperative behavior, i.e. recontraction between agents. (For a detailed discussion of the recontracting principle we refer to Greenberg (1986), Kaneko and Wooders (1986a, b), and Hammond, Kaneko and Wooders (1987).)

Unfortunately most existing measure theoretic models of perfect competitive economies are economically inconsistent, i.e. if one performs a "feed-back" from the model to its economic foundations (such as the primitive concept), then there arises an inconsistency. We will show this for some important models.

There are two primitive concepts used until now in modelling perfect competitive economies. The first one is the individual agent, i.e. an economic subject with individual characteristics only.

In his model **Aumann** (1964, 1966) defines an agent as an infinitely small set of positive measure in the continuum. This setting is however inconsistent with the notion of perfect competition itself and the Walras equilibrium as used in that model. (See also Hammond, Kaneko and Wooders (1987) for further discussion.)

Hildenbrand (1974, 1982) and **Shitovitz** (1973) define an agent as a single point in a measure space. This leaves however an inconsistency with the definition of the core, especially the fact that this particular primitive concept for the model allows all subsets of agents to recontract. (For a more elaborate discussion we refer to Hammond, Kaneko and Wooders (1987).)

The most important contribution to overcome these problems is the limit-approach in which a continuum economy is seen as the abstract limit of a sequence of finite, but growing, economies. (See **Bewley** (1973) and **Hildenbrand** (1974, 1982, and 1986).) However as Weiss (1981) and Armstrong (1985) argue, this approach is also inconsistent with the underlying notions. **Weiss** (1981) formulates an alternative by adjusting the measure theoretic notions of the model, but he is not able to build an economically consistent model, mainly because the measure theoretic notions become too weak.

The best answer to these conceptual problems is given by the f-core approach as developed in **Kaneko and Wooders** (1986a, 1986b) and **Hammond, Kaneko and Wooders** (1987). Although this approach gives new insights into the nature of the Aumann-Hildenbrand model, it is constructed on economically unrealistic assumptions. (By only allowing finite coalitions to recontract, the model does not recognize the existence of relatively large coalitions such as unions. This is confirmed by Kaneko and Wooders (1986b) in which it is shown that the f-core setting is the limit of situations in which only relatively small coalitions are allowed to recontract.)

The second class of models is based on the coalition as the primitive concept. Its use is justified by the observation that, in a case of perfect competition, only coalitions can influence the economic processes. The full consequence of accepting the coalition as primitive concept of the model is presented in **Armstrong and Richter** (1984, 1986) and **Armstrong** (1985). They totally abstract from the individual agent, and as a consequence only deal with **cooperative behavior**. Although their modelling is logically consistent, it is not possible to define a proper Walrasian equilibrium concept in this setting. Therefore the famous equality between Walras allocations and the core reduces to a more technical result. This is quite unsatisfactory.

In our opinion the major cause of these problems is the too rigourous application of classical measure theory, i.e. measure theory based on a σ-algebra. In this paper we present a new framework for this kind of models which is based on non-classical measure theory, based on a semi-ring instead of a σ-algebra.

This enables us to build the model on a new primitive concept: a coalitional structure of agents. It represents a class of interacting, and therefore social agents. Thus we do not deal with individual agents, i.e. with individual characteristics only, but with agents with individual as well as social characteristics. Thus we may consider an agent to be an individual or label combined with a set of individual characteristics and a set of social characteristics. (The reader should not confuse our notion of "coalitional structure" with the concept of "coalition structure" such as discussed in Aumann and Dreze (1974).)

In our model the set of individual characteristics of an agent consists of a preference relation on the set of attainable commodity bundles and an endowment. In future models this may be extended by adding some production technology or capacity.

The social characteristics of an agent are described by a social environment in which the agent operates. We can describe this environment by a class of coalitions of which the agent is a member. Since the social environment is the outcome of social interaction between all agents in the economy, and in real-life economies we observe several, different forms of interaction between agents, we have to **construct** a differentiated model of the social environment of the agent. We do this by constructing a differentiated structure of coalitions on the set of individuals, and thus we get a differentiated coalition concept.

The first step of this process is to construct a class of primitive coalitions on the set of individuals, which describes the most basic, and therefore strongest, form of interaction between agents. A primitive coalition is defined as a group of agents which plays a **structural** rôle in the economic processes. (Examples are enterprises, unions, political parties, families, etc.) We remark that it is evident that an agent can be a member of several primitive coalitions. In our static models we assume that these primitive coalitions can act as entities. As mentioned above, in our opinion this class cannot be described satisfactory by a σ-algebra of sets, since this would put too high a claim on the concept of primitive coalition. Therefore we assume that the class of primitive coalitions is a semi-ring on the set of individuals. (In a subsequent paper by Ruys and Gilles (forthcoming) it will be shown that this assumption is quite plausible. In that paper, the underlying concepts of interaction are described more thoroughly.)

We now assume that all other forms of interaction are based on the class of all agents in the economy, who are interacting in the described, primitive way. After introduction of a normalised σ-additive setfunction on the semi-ring of primitive coalitions, which denotes a size to these coalitions, we get a **non-classical** measure space, which fully describes this primitive concept of the model and therefore will be called a **coalitional structure of agents**.

The next step in the construction or extension process leads to the definition of the class of realizable coalitions. These coalitions are based on cooperation between a finite number of primitive coalitions. This kind of coalitions is quite realistic because it is based on a very simple form of cooperation, namely that of between primitive coalitions, i.e. structural groups of agents.

The final step in the extension process is to describe all feasible coalitions. This feasibility condition is with respect to the coalitional structure as defined above, and thus coincides with the measurability condition on measure spaces. This definition is justified by the character of the coalitional structure, i.e. the agent is seen in context of his/her environment, and by the assumption that the economic process is only influenced by coalitions with positive measure. (This last assumption is a direct consequence of adopting a measure into the description of the primitive concept of the model.)

Now all feasible coalitions are just the measurable sets of individuals with respect to the coalitional structure, and thus they form a σ-algebra. Another consequence is that allocations have to be feasible to the structure of the economy, and thus they are just the integrable functions from the set of individuals to the commodity space.

The definition of the Walras equilibrium, first used by Aumann (1964), can be applied in our model. It is consistent with a coalitional structure as the primitive concept of the model and the feasibility condition with respect to allocations, which is also justifiable for the non-cooperative case. (The agent always acts in the context of his/her environment.)

Since the model is thus build on a **differentiated coalition concept** we have to reconsider the definition of the core. The approach offered by Greenberg (1986) gives us a good insight into the nature of the recontracting principle: A coalition is able to "leave" the economy (the "room" as Greenberg puts it) and to redistribute its resources over the members of the coalition. This insight leads us to consider several core-like equilibrium concepts:

- The adoption of the original core as defined by Aumann (1964) in our setting is equivalent to the acceptance that **all feasible** coalitions with positive size are able to "leave the room". (As a consequence of the primitive concept, coalitions without size have no influence on the economy, i.e. are negligible; The primitive coalitions with size are the basic groups with direct influence on the economy.) However it is unlikely that all feasible coalitions actually are able to act in this way. In some cases the formation costs will be very high, and example 4.1 shows that there also are "structural" objections against the acceptance of this equilibrium concept.
- To overcome these objections we restrict the class of coalitions which are able to block, i.e. "leave" the economy, to the class of realizable coalitions with positive size. This leads to an extension of the original core which we will call the semi-core.
- The next step is to reduce the concept of recontracting: We only allow a finite number of disjoint primitive coalitions to recontract by **contract**, i.e. primitive coalitions block some proposed allocation by a contract. (In our case contracts are allocations in which primitive coalitions are treated as if they are a single agent, i.e. it is a bargain between disjoint primitive coalitions. Within a participating coalition all members are treated equal, and thus contracts satisfy an equal treatment property in resources.) This very rudimental form of blocking leads to an even larger extension of the original core which we will call the contract-core. It is only used to show how far the contracting principle can be reduced.

Usually the semi- and contract-core are larger than the core, but if the primitive coalitional structure is fine with respect to the feasible coalitions, and the economy satisfies some, more technical, property then all three core-like concepts are equal. If additionally the economy is atomless, which is the usual case, we can even abstract from the technical property to establish equality between the set of Walras allocations and the semi-core. The finess property is called conformability, and the technical property is called the strong core property, or SCP. Conformability expresses that the primitive structure in the economy is conform to, or dense in, the σ-algebra structure of all feasible coalitions.

We mention that economies based on a continuum of agents always are conformable and atomless, and therefore the semi-core is equal to the core.

Finally we remark that the theory developed in this paper is only based on measure-theoretic notions. We do not introduce any topological concepts in the definition of the economy, as is done by Hildenbrand (1974), and by Hammond, Kaneko and Wooders (1987) in modelling the f-core approach. There is neither an economic reason nor a justification to introduce such topological concepts.

In section 2 of this paper we develop the model based on the primitive concept described above. Furthermore we introduce the notions of Walras equilibrium, core, semi-core, and contract-core. In the third section we present the main equivalence results. In the next section we give some examples to show that conformability cannot be missed as a condition to the results. Finally we present some conclusions in a separate section.

Our model is mainly based on non-classical measure theory which is based on a semi-ring instead of a σ-algebra. For the clasical measure theory we refer to textbooks of de Barra (1981), Halmos (1950), or Jacobs (1978). Non-classical measure theory presented in a classical setting can be found in Zaanen (1967), and Aliprantis and Burkinshaw (1981). A modern and very general approach can be found in Janssen and van der Steen (1984), which is the main reference in this paper.

2. THE MODEL

In this section we will develop some nonclassical measure theory based on a semi-ring instead of a σ-algebra of subsets. We develop this theory in economic terms, especially from the point of view of the new primitive concept, the agent as a person with individual as well as social characteristics. For a detailed and more general approach in mathematical terms only, we refer to Janssen and van der Steen (1984), Zaanen (1967), or Aliprantis and Burkinshaw (1981).

After the development of the model in these nonclassical measure theoretic terms we present, in this measure theoretic context, some economic equilibrium concepts for situations with non-cooperative as well as with cooperative behavior. In the next section we wil give some results concerning the equivalence between these concepts.

As mentioned in the introduction, we deal with a sophisticated primitive concept: The agent is defined as an economic subject in a coalitional environment, i.e. as a **social agent**. In all his activities he takes this environment, the coalitional structure, as given and thus the results of these actions have to be feasible for this structure.

First we define a coalitional structure of agents. Let A be a set of labels or individuals. On this set we define a <u>semi-ring of primitive coalitions</u> as a collection Γ of subsets of A such that

$$\emptyset \in \Gamma \qquad ;$$

$$\forall\, E,F \in \Gamma : E \cap F \;,\; E\backslash F \in \Omega(\Gamma),$$

where $\Omega(\Gamma) := \{ \bigcup_{n=1}^{N} E_n \mid N \in \mathbb{N},\ E_n \in \Gamma, \text{ pairwise disjoint } (1 \leq n \leq N) \}$. If Γ is a semi-ring, then $\Omega(\Gamma)$ is the <u>ring</u> generated by Γ.

The condition that is put on the primitive coalitions is that all differences and intersections can be partitioned into a finite collection of primitive coalitions. Thus we assume that the social environment consists of enough small coalitions on which it can be build.

Given a set of individuals A and a semi-ring of primitive coalitions Γ on A, we define a <u>measure</u> as a setfunction $\mu : \Gamma \longrightarrow \mathbb{R}_+$ such that

$$\mu(\emptyset) = 0 \qquad ;$$

$$\sup \{ \sum_{n=1}^{\infty} \mu(E_n) \mid E_n \in \Gamma \ (n \in \mathbb{N}), \text{ pairwise disjoint } \} = 1 \qquad ;$$

If $E_n \in \Gamma$ $(n \in \mathbb{N})$ are pairwise disjoint primitive coalitions and

$E := \bigcup_{n=1}^{\infty} E_n \in \Gamma$, then $\mu(E) = \sum_{n=1}^{\infty} \mu(E_n)$.

This last property is usually called σ-additivity. A measure assigns a size to every primitive coalition in Γ.

Now a <u>coalitional structure of agents</u> is defined as a triple (A,Γ,μ) in which A is a set of individuals, Γ a semi-ring of primitive coalitions on A and μ a measure on (A,Γ).

Now an <u>agent</u> is **not** defined as a point in A such as in Hildenbrand (1974, 1982) or Hammond, Kaneko and Wooders (1987), but as an element in the coalitional structure. Thus an agent is an individual $a \in A$ **in the context of** his/her environment. This can be expressed by defining the social characteristics of an agent as a pair (a,Γ_a) with $a \in A$ being the label of the agent, and $\Gamma_a := \{ E \in \Gamma \mid a \in E \}$ being the social environment (or characteristic) of the agent. (In a subsequent paper we will use this expression frequently in the discussion of certain properties of coalitional structures.)

This social environment has essentially two effects on the behavior of the agent. Firstly, in his **non-cooperative behavior** the agent is influenced by his/her environment, because he/she is embedded in this coalitional structure. The only feasible assignments of commodities are those which are feasible with respect to the structure. (This feasibility condition is defined next.)

Secondly, in **cooperative behavior** the agent only achieves something when he/she cooperates with other agents. In the first place he/she operates in "obvious" coalitions which have a structural character. These "first order" coalitions are described by the class of primitive coalitions. We assume that this class forms a semi-ring and its members are able to operate in the economic process as an entity, i.e. are able to cooperate in the sense that, in a bargaining process, such a primitive coalition can act as if it was a single agent. Secondly, there exist groups of agents which are feasible for the primitive structure: These are the temporarily formed coalitions, i.e. formed by cooperation of primitive coalitions or simply formed by interaction between agents. (In our setting "given their social characteristic or environment".)

The economic justification of the fact that these "first order" or primitive coalitions form a semi-ring on the set of persons can be found in the forthcoming paper by Ruys and Gilles. There it will be shown that if interaction is based on some specific form of communication this class of primitive coalitions is a semi-ring on the set of individuals.

Based on the considerations mentioned above we have to seek for forms of cooperation between agents within the setting of the coalitional structure. The simplest extension of the primitive coalition is the realizable coalition, which is a member of the ring $\Omega(\Gamma)$ generated by the semi-ring of primitive coalitions. Thus a realizable coalition is the union of a finite number of pairwise disjoint primitive coalitions. It is quite obvious that this kind of cooperation between primitive coalition is realistic. As an example we mention cooperation between enterprises to form cartels.

It is clear that, based on the properties of primitive coalitions, the simplest assignment of goods is a contract between primitive coalitions. In this setting a contract is defined as a function $t\colon A \longrightarrow \mathbb{R}^{\ell}_{+}$, where $\mathbb{R}^{\ell}_{+}$ is the commodity space, such that

$$t(a) = \sum_{n=1}^{N} c_n \cdot \chi_{E_n}(a), \quad a \in A,$$

with $N \in \mathbb{N}$, $c_n \in \mathbb{R}^{\ell}_{+}$, and $E_n \in \Gamma$ pairwise disjoint ($1 \leq n \leq N$). (By χ_E we denote the indicator function of the set E.)

In mathematical terms t is a stepfunction. As we can see from this definition, contracts are based on the principle of equal treatment, i.e. within a participating coalition all members are treated equal. Thus in a contract the participating primitive coalition acts as an entity. Also it is clear that the set of agents, whom participate in the contract, is a realizable coalition.

Let $t = \sum_{n=1}^{N} c_n \cdot \chi_{E_n}$ be contract then

$$\int t \, d\mu := \sum_{n=1}^{N} c_n \cdot \mu(E_n)$$

is defined as the <u>mean contract</u>. It is an expression for the mean assignment of commodities in the contract to the participating agents. (This expression exists because of the finiteness assumption made in the definition of the coalitional structure (A,Γ,μ).)

Now we are able to define an <u>assignment of commodities</u> as a function $f\colon A \longrightarrow \mathbb{R}_+^{\ell}$ which is the pointwise limit of some sequence of contracts (t_n) such that $\int t_n \, d\mu$ converges to some (finite) limit, denoted by $\int f \, d\mu$. This limit is an expression for the <u>mean assignment</u> belonging to f.

Thus in some sense an assignment follows a feasibility condition with respect to the coalitional structure. As argued in the introduction, this feasibility condition is quite natural, because it is a direct consequence of the acceptance of the primitive concept: The agents are embedded in the structure, and thus all assignments have to fit with respect to this structure, i.e. have to be feasible (with respect to this structure). In mathematics this feasibility condition is called <u>measurability</u>. With it we are able to justify that we only have to consider a certain class of functions on A as the collection of assignments.

A special class of assignments are the assignments of memberships: A set $E \subset A$ is a <u>coalition</u> if χ_E is a feasible assignment (of membershipcards). The collection of all coalitions is denoted by Σ and it forms a σ-algebra of subsets of A which is complete with respect to μ. Thus it is clear that the σ-algebra Σ describes **all feasible coalitions** in the coalitional structure. For the definition of the equilibrium concepts in an economy we only have to take this collection into account.

Before we are able to define allocations we have to describe some measure theoretic notions. Let (A,Γ,μ) be a coalitional structure and let the commodity space be given by $\mathbb{R}^{\ell}_{+}$.

The measure μ on Γ can be extended to Σ. For every $E \in \Sigma$ define $\hat{\mu}(E) := \int \chi_E \, d\mu$ then $\hat{\mu}$ is a σ-additive setfunction on Σ which coincides with μ on Γ. As usual we denote this extension also by μ. We can see that μ assigns to every coalition its mean membership, i.e. the size of that coalition.

If $E \in \Sigma$ and $\mu(E) = 0$ then E is called a null-set. Null-sets of agents are so small that they do not have any measurable influence on the economic processes. In other words, null-sets will not be noticed when they leave the room, and thus are negligible. (The economic power is based on the size of the coalition, and does not originate from the individual only, but from the totality of **individual and social characteristics** of the agents.) This last observation is a direct consequence of the fact that size (i.e. the measure μ) is an important part of the social environment.

If f and g are assignments, then we define f = g a.e. if $f(a) = g(a)$ for all $a \in A \setminus E$, with $E \in \Sigma$ and $\mu(E) = 0$, i.e. E is a null-set. Thus on the class of assignments we can define an equivalence relation. The equivalence classes of this relation are denoted by

$$[f] := \{ g \mid g \text{ is an assignment and } f = g \text{ a.e. } \},$$

$$\text{and } \int [f] \, d\mu := \int g \, d\mu, \text{ with } g \in [f] \text{ arbitrary.}$$

(In the definitions above we naturally assume that f is an assignment.)

Thus f = g a.e. means that f and g are **de facto** equal assignments of commodities, i.e. the difference is negligible. Therefore we only have to study these equivalence classes. In the sequel we denote an equivalence class [f] simply by f and define $L(\mu,\mathbb{R}^{\ell}_{+})$ to be the set of all equivalence classes with respect to the coalitional structure (A,Γ,μ) and commodity space $\mathbb{R}^{\ell}_{+}$. We call $f \in L(\mu,\mathbb{R}^{\ell}_{+})$ an allocation and $\int f \, d\mu$ is denoted to be the mean allocation.

Now we have extended the coalitional structure in two steps. The first step leads to **contracts** and **realizable coalitions**. (It is the simplest extension that is possible.) The second step leads to the boundaries of what is feasible: **Allocations** and **coalitions**. (Note that there is no economic difference between assignments and allocations, because in the coalitional structure one naturally abstracts from negligible differences.)

Also note that (A,Σ,μ) is an extension of (A,Γ,μ) and that it is a classical measure space. Most authors, as mentioned in the introduction, however depart from a classical measure space. Thus we may conclude that the extension process, as defined above, leads to the same tools as used by most authors.

Now we are able to define an exchange economy:

2.1 Definition.

An exchange economy is defined as the triple $E_x := [(A,\Gamma,\mu), \{\succ_a\}_{a \in A}, w]$, where

(A,Γ,μ) is a coalitional structure such that $\mu(A) = 1$ (Normalisation of economic size) ;

$\{\succ_a\}_{a \in A}$ is a set of preference relations on the commodity space $\mathbb{R}_+^\ell$, such that for every $a \in A$, $\succ_a$ is irreflexive and transitive ;

$w \in L(\mu, \mathbb{R}_+^\ell)$ is the initial allocation or endowment with $\int w \, d\mu \gg 0$.

Thus an exchange economy is defined in context to the primitive concept, i.e. the agent in a coalitional structure. Again we stress that in an exchange economy we only have to observe the class of allocations, because this fully describes what is feasible with respect to the coalitional structure. From this argument we introduce the traditional economic feasibility condition:

2.2 Definition.

Let E_x be an exchange economy.

A feasible allocation for E_x is an allocation $f \in L(\mu,\mathbb{R}_+^\ell)$ such that

$\int f \, d\mu \leq \int w \, d\mu$.

The set of all feasible allocations for E_x is denoted by $\mathbf{S(E_x)}$.

The economic feasibility condition on allocations only says that it is not possible to allocate more than there initially exists in the economy.

The traditional equilibrium concept for the case of non-cooperative behavior is the Walras equilibrium, which is also consistent with the primitive concept such as defined above. In the case of non-cooperative behavior the coalitional structure only plays a latent rôle. Because of this rôle we only have to consider allocations such as defined above, which is done also in the definition originally given by Aumann (1964):

2.3 Definition.

Let E_x be an exchange economy.

A Walras equilibrium is defined as the pair (f,p) with $f \in S(E_x)$ and $p \in \Delta$ such that, a.e. on A, $f(a)$ is a maximal element for $\succ_a$ in

$$\{ x \in \mathbb{R}_+^{\ell} \mid p.x \leq p.w(a) \},$$

where Δ is the unit simplex in $\mathbb{R}^{\ell}$. Usually f is called a Walras allocation.
Notation: $f \in W(E_x)$.

As an interpretation of this notion we can say that in a Walras equilibrium every agent, except a negligible set, chooses an optimal bundle within his budgetset. The perfect competition assumption garantees the fact that every agent acts as a pricetaker.

Now we have defined the equilibrium concept for the case of non-cooperative behavior, we switch to the case of cooperative behavior, in our setting the recontracting principle. The proposed solution concepts for this case are the main topic of this paper. Before we define them, we have to define some forms of recontraction:

2.4 Definition.

Let E_x be an exchange economy.

(a) Let $E \in \Sigma$ be a coalition and $f \in L(\mu, \mathbb{R}^{\ell}_{+})$ be an allocation. Then E is able to improve upon f if

$\mu(E) > 0$ and

there exists an allocation $g \in L(\mu, \mathbb{R}^{\ell}_{+})$ such that
$g(a) \succ_a f(a)$, a.e. on E, and $\int_E g \, d\mu \leq \int_E w \, d\mu$.

(b) Let $f \in L(\mu, \mathbb{R}^{\ell}_{+})$ be an allocation. Then f can be blocked if there exists a coalition $E \in \Sigma$ which is able to improve upon f.

(c) Let $f \in L(\mu, \mathbb{R}^{\ell}_{+})$ be an allocation. Then f is blocked strongly if there exists a coalition $E \in \Sigma$ which is able to improve upon f by choosing $g \in L(\mu, \mathbb{R}^{\ell}_{+})$ such that there exists a number $\varepsilon > 0$ such that for every allocation $h \in L(\mu, \mathbb{R}^{\ell}_{+})$:

$$\|g(a)-h(a)\| < \varepsilon, \; a \in E \implies h(a) \succ_a f(a), \; a \in E.$$

(d) Let $f \in L(\mu, \mathbb{R}^{\ell}_{+})$ be an allocation. Then f is said to be blocked by a pseudo-contract if there exists a simple function $g := \sum_{n=1}^{N} c_n \cdot \chi_{E_n}$, with $c_n \in \mathbb{R}^{\ell}_{+}$ and $E_n \in \Sigma$ $(1 \leq n \leq N)$ pairwise disjoint such that

$$c_n \succ_a f(a), \; a \in E_n \qquad (1 \leq n \leq N) \; ;$$

$$\sum_{n=1}^{N} c_n \cdot \mu(E_n) \leq \int_{\bigcup_{n=1}^{N} E_n} w \, d\mu .$$

(e) Let $f \in L(\mu, \mathbb{R}^{\ell}_{+})$ be an allocation. Then f is said to be blocked by a contract if there exists a contract $t := \sum_{n=1}^{N} c_n \cdot \chi_{E_n}$ with $c_n \in \mathbb{R}^{\ell}_{+}$ and

$E_n \in \Gamma$ $(1 \leq n \leq N)$ pairwise disjoint such that

$$c_n \succ_a f(a) \;,\; a \in E_n \qquad (1 \leq n \leq N) \;;$$

$$\sum_{n=1}^{N} c_n \cdot \mu(E_n) \leq \int_{\bigcup_{n=1}^{N} E_n} w \, d\mu \;.$$

In the previous definition we have given four forms of blocking or recontraction. The first one is the basic recontracting principle as used by many authors. It simply says that a coalition is able to threat to block a proposed allocation if it is noticed, i.e. has enough size, and all its members are willing to join if the threat has to be executed. (We stress that all forms of recontraction basicly are threats.)

Based on this general form of recontraction, we define some other forms. Strong blocking and blocking by a pseudo-contract are more technical forms, since these have no interesting economic features. However blocking by a contract has some additional features: it reduces the threat to a very rudimental level, and thus it becomes very practical.

Next we construct some solution concepts for the cooperative case which are based on some recontracting principle, i.e. on forms of blocking as defined above. Only three of these concepts are non-technical. After the definition we will discuss several aspects of these equilibrium concepts in case of cooperative behavior.

2.5 Definition.

Let E_x be an exchange economy.

(a) The core of E_x is defined as the set

$$C(E_x) := \{\; f \in S(E_x) \;|\; f \text{ cannot be blocked } \}.$$

(b) The semi-core of E_x is defined as the set

$$\tilde{C}(E_x) := \{\; f \in S(E_x) \;|\; \text{There is no realizable coalition } E \in \Omega(\Gamma) \text{ which is able to improve upon } f \;\}.$$

(c) The contract-core of E_x is defined as the set

$$B(E_x) := \{ f \in S(E_x) \mid f \text{ cannot be blocked by a contract } \}.$$

(d) Further we define the following sets:

$$\tilde{B}(E_x) := \{ f \in S(E_x) \mid f \text{ cannot be blocked by a pseudo-contract } \}$$

$$CS(E_x) := \{ f \in S(E_x) \mid f \text{ cannot be blocked strongly } \}.$$

From the differentiated coalition concept, as introduced in our model, we derive the different core-like equilibrium concepts as defined above. The basic consideration in this set-up is, that primitive coalitions are the basic forces in the recontracting processes. Secondly, we observe that non-realizable coalitions only have a theoretical meaning in the recontracting processes: They are (structurally) feasible, but it is not likely that this kind of coalitions actually will form. (There are high search costs to locate agents who want to join such a particular coalition.) Therefore we conclude that only realizable coalitions are realistic forces in the recontracting processes in an economy.

Now the core is based on blocking by all feasible coalitions, although, as argued above, non-realizable coalitions are not likely to form. The semi-core is the result of a first restriction of possible blocking coalitions: Based on purely structural considerations we reduce to realizable coalitions. The main consideration is that it is (structurally) **not justifiable** that all feasible coalitions are able to block, especially since this is **inconsistent** with the recontracting principle itself. (Namely there is only the threat to leave the economy. Not all (feasible) coalitions are able to execute this threat by actually leaving the economy and act on their own. We refer to example 4.1 for a further discussion.)

Thus the semi-core is a first step in the reduction of blocking coalitions: Only coalitions formed out of cooperation between a finite number of primitive coalitions are able to make realistic threats. (As discussed Ruys and Gilles (forthcoming) for certain economies this concept of recontraction is still too crude.)

The next step is to reduce the recontracting principle to a minimum with respect to the coalitional structure, by considering blocking by contracts only. A contract is directly observable: a primitive coalition acts as a single agent in the bargaining process to form a contract. A contract satisfies an equal treatment property. Therefore the core-like concept based on contracts has its own normative features: it gives us a further insight into the nature of the core on normative grounds if we can establish a connection between these two concepts.

Finally the concepts based on pseudo-contracts and strong blocking are only of secondary value. They give us the tools to connect the main core-like equilibrium concepts: core, semi-core, and contract-core.

We conclude this section with a first result on these solution concepts as presented in definition 2.5. In the next section we will give the main equivalence results.

2.6 Proposition.

Let E_x be an exchange economy. Then

(1) $W(E_x) \subset C(E_x) \subset \tilde{C}(E_x) \subset B(E_x)$

(2) $W(E_x) \subset C(E_x) \subset \tilde{B}(E_x) \subset B(E_x)$

(3) $W(E_x) \subset C(E_x) \subset CS(E_x)$.

Proof.

The major inclusion $W(E_x) \subset C(E_x)$ can be found as proposition 1, Hildenbrand (1974), p. 131. The other inclusions follow directly from the definition.

□

The next picture gives us a view on the relations between all proposed core-like solution concepts.

$$
\begin{tikzcd}
 & CS(E_x) & \\
W(E_x) \arrow[r] & C(E_x) \arrow[u] \arrow[r] \arrow[d] & \tilde{C}(E_x) \arrow[d] \\
 & \tilde{B}(E_x) \arrow[r] & B(E_x)
\end{tikzcd}
$$

(In this picture the inclusions are given by arrows.)

3. THE RESULTS

In this section we will state some equivalence results for the equilibrium concepts such as developed in the previous section. First we prove that under quite natural assumptions all concepts under consideration are not empty, and thus our investigation is not in vain. Secondly we give the conditions under which the core is equal to the semi-core and the contract-core, i.e. blocking by contracts is equivalent to normal blocking.

Next we state an extension of the Aumann-equivalence theorem which describes the equality between core, semi-core, and the set of Walras allocations. Finally we will present a summary of all equivalence statements as presented in this paper, especially in this section.

First we formulate an existence result. Under the assumptions stated below we prove that there is a Walras equilibrium and therefore the set of Walras allocations is non-empty.

3.1 Assumption.

Let E_x be an exchange economy. We assume that:

(a) For every $a \in A$, $\succ_a$ is monotone, and if a belongs to an atom of (A,Γ,μ), then additionally $\succ_a$ is convex. [An atom of (A,Γ,μ) is a coalition $E \in \Sigma$ such that $\mu(E) > 0$ and if $E \supset F \in \Sigma$, then $\mu(F) = 0$ or $\mu(E) = \mu(F)$.]

(b) For all $y \in \mathbb{R}^{\ell}_{+}$, $\{ (a,x) \mid y \succ_a x \} \in \Sigma \otimes \mathcal{B}(\mathbb{R}^{\ell}_{+})$, where $\mathcal{B}(\mathbb{R}^{\ell}_{+})$ is the σ-algebra of all Borelsets of $\mathbb{R}^{\ell}_{+}$, the positive orthant of the ℓ-dimensional Euclidean space.

For an interpretation of this assumption we can say that part (a) is standard. The assumptions on the preferences as stated in 3.1 (a) are widely used in the literature on general equilibrium theory.

Part (b) consists of a **measurability condition** on the preferences. In our setting we are able to interpret this condition quite well: it simply says that the preferences are **feasible** with respect to the coalitional structure of the economy in the sense of the previous section. Thus, interaction between agents and their environment leads to adaption of individual characteristics, in this case preferences, with respect to the characteristics of the agents in that environment. One of the social characteristics of the agents is that interaction with the social environment, and thus the measurability condition on preferences is quite natural in the context of the primitive concept of the model.

3.2 Lemma.

Let E_x be an exchange economy which follows assumption 3.1. Then there exists a Walras equilibrium $(f^*, p^*) \in S(E_x) \times \Delta$ with $p^* \gg 0$.

Proof:

Since our conditions are slightly different from the ones used in the literature, we have to prove the assertion in a more direct way, which is an application of the technique used by Aumann (1964).

Let $p \in \Delta$, then for every $a \in A$ we define:

$$B(a,p) := \{ x \in \mathbb{R}^{\ell}_{+} \mid p.x \leq p.w(a) \}.$$

$$\Phi(a,p) := \{ x \in B(a,p) \mid \nexists\, y \in B(a,p) \colon y \succ_a x \}.$$

Now assume $p \gg 0$, then $B(.,p)$ has a measurable graph since $w \in L(\mu, \mathbb{R}^{\ell}_{+})$. (This is easily concluded by direct application of the definitions of measurability as given in Castaing and Valadier (1977).) Now we prove that if $p \gg 0$, then $\Phi(.,p)$ also has a measurable graph:

$$\begin{aligned}\text{Graph}(\Phi(.,p)) &= \{ (a,x) \mid x \in \Phi(a,p) \} = \\ &= \{ (a,x) \mid x \in B(a,p) \} \setminus \{ (a,x) \mid \exists \text{ rational } r \in B(a,p) \colon r \succ_a x \},\end{aligned}$$

because $\succ_a$ is assumed to be continuous $(a \in A)$. Since $B(.,p)$ is measurable we only have to check the measurability of the second set:

$$\{ (a,x) \mid \exists \text{ rational } r \in B(a,p) : r \succ_a x \} =$$
$$= \bigcup_{r \in \mathbb{Q}^{\ell}} [\{ (a,x) \mid r \in B(a,p) \} \cap \{ (a,x) \mid r \succ_a x \}] =$$
$$= \bigcup_{r \in \mathbb{Q}^{\ell}} [(B^{-}(\{r\}) \times \mathbb{R}^{\ell}_{+}) \cap \{ (a,x) \mid r \succ_a x \}],$$

and from assumption 3.1(b) we see that this set is measurable.

Now we have established the measurability of $\Phi(.,p)$ for $p \gg 0$, and thus we are able to apply the proof of Debreu (1982). He uses theorem 2 of Hildenbrand (1974, p. 151) and therefore checks properties (ii) - (iv) of proposition 3 in Hildenbrand (1974, p. 149). In that case Debreu only uses assumption 3.1(a) and the measurability of $\Phi(.,p)$ for $p \gg 0$.

□

Now we are able to state the first main theorem. Before we do so, we remind the reader that $\sigma(\Gamma)$ is the σ-algebra generated by the semi-ring of primitive coalitions Γ, i.e. $\sigma(\Gamma)$ is the **smallest** σ-algebra that contains Γ. It is clear that $\sigma(\Gamma) \subset \Sigma$.

3.3 Theorem.

Let E_x be an exchange economy, and let $f \in S(E_x)$ be a feasible allocation for E_x. Then:

(a) $f \in C(E_x)$ if and only if there does not exist a coalition $E \in \sigma(\Gamma)$ which is able to improve upon f.

(b) $C(E_x) \subset \tilde{B}(E_x) \subset CS(E_x)$

(c) If E_x follows assumption 3.1, then $\emptyset \neq W(E_x) \subset C(E_x)$.

Proof:

For the proof of parts (a) and (b) we refer to the appendix. Part (c) is a simple application of lemma 3.2 and proposition 2.6.

□

The theorem is important for a first interpretation of the model. The first part of the theorem states that the definition of the core as given in the previous section is equivalent to the definition that most authors use. (Usually one takes a certain σ-algebra for Γ and does not extend to Σ as is done in our setting.) We conclude that our model, with its differentiated coalition concept, can deal with the core as used in the literature on models **without** a differentiated coalition concept.

The second part of the theorem shows that strong blocking is stronger than blocking by a pseudo-contract. Although the result has no direct economic meaning it gives us a tool in the discussion of core-like equilibrium concepts as presented in the previous section.

Finally, the third part of the theorem is a restatement of the previous result, lemma 3.2: Under quite acceptable and plausible assumptions all concepts considered in this paper are non-empty.

We proceed by formulating a condition for equivalence of the core and the more technical concept which is a result of blocking by a pseudo-contract.

3.4 Definition.

An exchange economy E_x is said to satisfy the Strong Core Property, or SCP, if for every allocation $f \in L(\mu, \mathbb{R}^{\ell}_{+})$:

If f is blocked, then f can be blocked strongly.

3.5 Corollary.

If E_x is an exchange economy which has the Strong Core Property (SCP), then

$$C(E_x) = CS(E_x) = \tilde{B}(E_x).$$

Proof:

SCP actually states that $CS(E_x) \subset C(E_x)$, and thus from theorem 3.3(b) the assertion follows immediately.

□

In the next section we will present some examples which will show that for some exchange economies E_x which satisfy the Strong Core Property the following inequality holds: $C(E_x) \subsetneq \tilde{C}(E_x) \subsetneq B(E_x)$. (Thus then we may conclude that SCP on itself is <u>not</u> enough to garantee a meaningfull extension of Aumann's equivalence theorem.)

The Strong Core Property is not yet developed into full detail. It is not clear under which conditions it is valid, or even if it is valid for many economies with uncountable many agents. It is clear that further research is needed to solve this open problem.

Next we will give the additional condition under which the core is equal to the other core-like concepts as defined in the previous section:

3.6 Definition.

(a) A coalitional structure of agents (A,Γ,μ) is called <u>(inner) conformable</u> if for every $E \in \Sigma$:

$$\mu(E) = \sup \{ \mu(F) \mid E \supset F \in \Omega(\Gamma) \}.$$

(b) An exchange economy E_x is called <u>conformable</u> if it has an (inner) conformable coalitional structure of agents.

In the next section we will show in some examples that the conformability property is very strong, also for "normal" economies. In fact it states that the semi-ring of primitive coalitions is so rich that most properties with respect to Σ can be reduced (without any distortion) to $\Omega(\Gamma)$ and thus to Γ. In a subsequent paper we will show that the conformability condition is equivalent to the condition that the coalitional structure is flexible. (This property makes clear that many large economies do not satisfy this condition.)

It may be clear that the name "conformable" is choosen to express the density of the coalitional structure, relative to what is feasible in the economy. The primitive structure is "conform" the extended structure of all feasible coalitions, i.e. the primitive structure can be used instead of the extended structure. In the results in the sequel we actually will see that the properties with respect to the σ-algebra of feasible coalitions Σ simply carry over to the ring of realizable coalitions $\Omega(\Gamma)$ if the economy is conformable.

Now we are able to formulate the first of our main equivalence theorems:

3.7 Theorem.

Let E_x be a conformable exchange economy which satisfies SCP.

(a) $f \in S(E_x)$ can be blocked by a pseudo-contract if and only if f can be blocked by a contract.

(b) $C(E_x) = \tilde{C}(E_x) = B(E_x)$.

Proof:

For a proof of part (a) we refer to the appendix. Part (b) is an application of theorem 3.3 and part (a): If $f \notin C(E_x)$, then by SCP and corollary 3.5 it can be blocked by a pseudo-contract, and thus by part (a) of the theorem, it can be blocked by a contract. Now from the inclusions $C(E_x) \subset \tilde{C}(E_x) \subset B(E_x)$ as stated in proposition 2.6, we derive the equalities.

□

We conclude this section by examining the conditions under which there is an equality between the set of Walras-allocations and the semi-core. This is an extension of the famous result as established by Aumann (1964). He proved that under the condition that the economy has an atomless measure space of agents, $W(E_x) = C(E_x)$. (For a proof we refer to Hildenbrand (1974, 1982). For some other extensions we refer to Armstrong and Richter (1984), for the case with coalitions only, and to Shitovitz (1973) and Greenberg and Shitovitz (1986) for the extension to the case with large traders also. Finally we mention Hammond, Kaneko and Wooders (1987) for an extension to equality of the set of Walras allocations and the f-core.)

First we have to define an atomless economy:

3.8 Definition.

An exchange economy E_x is called <u>atomless</u> if for every (feasible) coalition $E \in \Sigma$ with size $\mu(E) > 0$, there exists a sub-coalition $E \supset F \in \Sigma$ such that

$$0 < \mu(F) < \mu(E).$$

The reduction of this property to the semi-ring of primitive coalitions Γ instead of the σ-algebra of coalitions Σ, and the connection of this property with the conformability condition is presented in a subsequent paper. In that paper we prove that atomlessness does not only describe perfect competition, but also implies pluriformity of the coalitional structure and the almost social uniqueness of an agent. In this paper we only analyse the consequence of this assumption for the equivalence between some core-like equilibrium concepts and the set of Walras allocations.

The next theorem is the second main theorem of this paper:

3.9 Theorem.

Let E_x be an exchange economy. Let for every $a \in A$, $\succ_a$ be monotone and measurable in the sense of assumption 3.1 (b).

If E_x is atomless and conformable, then

$$W(E_x) = C(E_x) = \tilde{C}(E_x) .$$

Proof:

From Aumann's equivalence theorem , as mentioned above, we know that for an atomless economy $W(E_x) = C(E_x) \subset \tilde{C}(E_x)$. (The last inclusion follows from proposition 2.6.) The proof that $\tilde{C}(E_x) \subset W(E_x)$ can be found in the appendix. Thus the theorem is proved.

□

From the definition of conformability and the proof of theorem 3.3(a) it is clear that, if one uses a σ-algebra for Γ, the coalitional structure of agents is (inner) conformable. Thus we see that all models as developed in the literature satisfy conformability. Therefore our second main theorem is a direct extension of Aumann's equivalence theorem as presented in Aumann (1964).

In a subsequent paper we will tackle the economic interpretation and consequences of the atomlessness and conformability conditions on a coalitional structure of agents. These major structural conditions not only garantee the equivalence of the major non-cooperative equilibrium concept, the Walras equilibrium, and the major non-cooperative equilibrium concept, the semi-core, but also express some economic notions such as perfect competition. In that subsequent paper we will discuss these economic features of coalitional structures which satisfy these conditions in full detail.

SUMMARY

In the following pictures we give a summary of all results of this section:

A - Without any conditions.

$$\begin{tikzcd} W(E_x) \arrow[r] & C(E_x) \arrow[r] \arrow[d] & \tilde{B}(E_x) \arrow[r] \arrow[d] & CS(E_x) \\ & \tilde{C}(E_x) \arrow[r] & B(E_x) & \end{tikzcd}$$

B - Under the Strong Core Property.

$$\begin{tikzcd} W(E_x) \arrow[r] & C(E_x) \arrow[r, leftrightarrow] \arrow[d] & \tilde{B}(E_x) \arrow[d] \\ & \tilde{C}(E_x) \arrow[r] & B(E_x) \end{tikzcd}$$

C - Under conformability and SCP.

$$\begin{tikzcd} W(E_x) \arrow[r] & C(E_x) \arrow[r, leftrightarrow] & \tilde{C}(E_x) \arrow[r, leftrightarrow] & B(E_x) \end{tikzcd}$$

D - Under conformability and atomlessness.

$$\begin{tikzcd} W(E_x) \arrow[r, leftrightarrow] & C(E_x) \arrow[r, leftrightarrow] & \tilde{C}(E_x) \arrow[r] & B(E_x) \end{tikzcd}$$

4. SOME EXAMPLES

In this section we give some examples to achieve a more detailed insight into the nature of the equivalence theorems 3.3 (and thus of corollary 3.5), theorem 3.7 and especially theorem 3.9. First we show that the Strong Core Property is not sufficient to obtain equality of the core and the semi-core, and thus of the core and the contract-core. Secondly we will show that normally also the contract-core is strictly larger than the semi-core.

We mention that the reduction of normal blocking to blocking by contracts is only a first step in deriving a more realistic model of the recontracting principle and thus of cooperative behavior in the setting of a general equilibrium model. In the subsequent paper by Ruys and Gilles (forthcoming) this will be shown by considering certain economies with some special features.

4.1 Example.

In this example we construct an **atomless** exchange economy E_x , which satisfies **SCP**, but is **not conformable**. We show that in this economy the core is strictly smaller than the semi-core, and therefore we show that in theorem 3.9 the conformability condition is not superfluous. We conclude the example with a natural economic interpretation of the specific features of the constructed economy. This interpretation also gives us a clear view on some aspects of the coalitional structure of agents as the primitive concept of the model.

We define E_x by choosing $\ell = 3$, i.e. there are three commodities in the economy, and the following coalitional structure:

$$A := [0,1)\times[0,1) \qquad ;$$

$$\Gamma := \{\ [a,b)\times[c,d)\ \mid\ a,b \in [0,1] \text{ with } a < b\ ;\ c = 0 \text{ or } c > .5 \text{ and If } c = 0 \text{ then } .5 < d \leq 1, \text{ else (If } c > .5) \ c < d \leq 1\ \} \cup \{\ \emptyset\ \}\ ;$$

If $E = [a,b)x[c,d) \in \Gamma$, then $\mu(E) := (b-a).(d-c)$ and of course by assumption $\mu(\emptyset) = 0$.

Easily it is verified that Γ is a semi-ring of primitive coalitions and that (A,Γ,μ) is a coalitional structure of agents. We conclude the construction of the economy E_x by choosing the endowment and the preferences:

$w \in L(\mu, \mathbb{R}^{\ell}_{+})$ is the endowment with

$$w(a) := \begin{cases} (2,0,0) & a \in E_1 \\ (0,4,0) & a \in E_2 \\ (0,0,4) & a \in E_3 \end{cases}$$

with $E_1 = [0,1)x[0,.5] \in \Sigma$, $E_2 = [0,1)x(.5,.75) \in \Sigma$ and finally $E_3 = A\backslash(E_1 \cup E_2) = [0,1)x[.75,1) \in \Gamma$.

Finally we define preferences by the following utility function on the commodity space $\mathbb{R}^3_+$. If $\underline{x} = (x_1,x_2,x_3) \in \mathbb{R}^3_+$, then

$$u(a,\underline{x}) := \begin{cases} x_3 & a \in E_1 \\ x_2 & a \in E_2 \\ x_1 & a \in E_3 \end{cases}$$

From the definition of the constructed economy E_x we derive the following conclusions:

- E_x is atomless, but not conformable.
- Preferences are continuous, monotone, and measurable, and thus assumption 3.1 is satisfied. Thus there exists a Walras equilibrium (by application of lemma 3.2), and so all core-like concepts under consideration are non-empty.
- w is a pseudo-contract, i.e. a simple function.
- Any $E_2 \supset F \in \Sigma$ with size $\mu(F) > 0$ has no incentive to improve upon w. The only coalitions which have an **incentive** to improve upon w are

those which can be written as $F := F^\circ \cup F^*$ with $E_1 \supset F^\circ \in \Sigma$, and $E_3 \supset F^* \in \Sigma$. Now $F \in \Sigma$ has the **power** to improve upon w if and only if $\mu(F^\circ) > 0$ as well as $\mu(F^*) > 0$.

- Next we show that this economy satisfies the Strong Core Property. Let $G \in \Sigma$ and we define $G_i := G \cap E_i$ $(i = 1,2,3.)$. Now $G = G_1 \cup G_2 \cup G_3$. Now let $f \in S(E_x)$ and define

$$\delta_1 := 2.\mu(G_1) - \int_{G_3} f_1 \, d\mu \quad ;$$

$$\delta_i := 4.\mu(G_i) - \int_{G_{4-i}} f_i \, d\mu \quad (i = 2,3).$$

Now it is easily checked that G is able to improve upon f if and only if $\delta_2 > 0$ **or** $\delta_1 > 0$ as well as $\delta_3 > 0$ under the condition that: if $\delta_2 = 0$, then $\mu(G_2) = 0$, and if $\delta_i = 0$ (i=1 or i=3), then $\mu(G_1) = \mu(G_3) = 0$. Now we define:

$$g(a) := (0,0,f_3(a)+\varepsilon_3) \qquad a \in G_1 \quad ;$$

$$g(a) := (0,f_2(a)+\varepsilon_2,0) \qquad a \in G_2 \quad ;$$

$$g(a) := (f_1(a)+\varepsilon_1,0,0) \qquad a \in G_3 \quad ;$$

where $\varepsilon_1 := \delta_1/\mu(G_3)$; $\varepsilon_3 := \delta_3/\mu(G_1)$ if $\delta_1, \delta_3 > 0$; and $\varepsilon_2 := \delta_2/\mu(G_2)$ if $\delta_2 > 0$. (Choose $\varepsilon_i = 0$ if $\delta_i = 0$.)

Thus if G is able to improve upon f, G can do it by choosing the allocation g. Now assume G is able to improve upon f. By choosing

$$\tilde{G} := \cup \{ G_i \mid \delta_i > 0 \} \quad \text{and}$$

$$\varepsilon := \min \{ \varepsilon_i \mid \delta_i > 0 \}/2.$$

Then we know that $\mu(\tilde{G}) > 0$ and $\varepsilon > 0$. It is clear that f can be blocked by $\tilde{G}$ if it chooses g. Now for $\tilde{G}$ every allocation $h \in L(\mu, \mathbb{R}^3_+)$

such that $|g(a)-h(a)| < \varepsilon.1_3$ $(a \in \tilde{G})$ also blocks f. Thus we conclude that E_x satisfies the Strong Core Property, SCP.

From the observations above we are able to conclude the following:

(A) The endowment w cannot be blocked by any realizable coalition $E \in \Omega(\Gamma)$, but there exist coalitions $E \in \Sigma$ which **are able to improve upon** w. Thus $w \in \tilde{C}(E_x)$, but $w \not\in C(E_x)$, and so we conclude that in the constructed economy $C(E_x) \subsetneq \tilde{C}(E_x)$.

(B) If $F := F^* \cup F^\circ \in \Sigma$, as defined above, is able to improve upon w, it can be done by choosing a pseudo-contract. (This a consequence of SCP.) Thus now we conclude that $w \not\in \tilde{B}(E_x)$, but that $w \in \tilde{C}(E_x) \subset B(E_x)$.

Next we will give an interpretation of this economy.

Assume E_x describes an economy with three main groups of agents: E_1 represents the class of laborers, E_2 is the class of managers/management workers, and finally E_3 describes the other agents, in this case we take them to be rentiers.

To complete the description we assume that commodity 1 is a composite service commodity, these are delivered by the laborers, commodity 2 is capital owned by the management, and commodity 3 is simply a composite consumption good or money. Now the laborers want to gain money, managers are delighted by controlling capital goods, while the other agents want to get serviced.

In view of the coalitional structure in the described economy, it is clear we assume that labor cannot influence the economic process without any guidance of some management. In all primitive coalitions in which labor is represented, there always is a group of managers to guide the coalition. (It may be clear that we allow the existence of unions, since a union has some professional managers at the top to represent labor in negotiations.)

We see that, in this economy, management has no incentive to improve upon the initial allocation of commodities, and thus they will not cooperate to any kind of blocking. Since labor certainly has incentives to block the endowment, it has to operate on its own. Theoretically this is feasible, but practically most uncertain.

We conclude that in this economy the structure is "stiff" in the sense that labor cannot really operate on its own. Thus, labor has no real power to threat to block unless a group of managers agrees. So it is realistic to have the endowment in some core-like equilibrium concept. (As we described previously: core-like equilibrium concepts are based on threats, and in this case labor cannot generate real threats on its own.)

We conclude that this example describes a situation in which we have to choose: if we consider **all theoretical threats**, we have to accept the **core** , and if we only consider **the realistic threats**, we have to accept the **semi-core**.

□

4.2 Example.

In this example we construct another exchange economy E_x that satisfies SCP, but does not satisfy conformability. For this economy we will show that the semi-core is strictly smaller than the contract-core. We actually show that there exists a primitive coalition which is able to improve upon the endowment, but that there does not exist a contract which can block the endowment. By this example we want to give some extra information with respect to blocking with contracts in contrast to other kinds of blocking.

We assume there are two commodities ($\ell = 2$). Now the exchange economy E_x is constructed as follows:

$A := [0,1)$

$\Gamma := \{ [a,b) \mid (a = 0 \text{ and } .5 < b \leq 1) \text{ or } .5 < a < b \leq 1 \} \cup \{\emptyset\}$

If $E = [a,b) \in \Gamma$, then $\mu(E) := b-a$ and naturally $\mu(\emptyset) = 0$.

Now (A,Γ,μ) is a coalitional structure of agents, but it is not atomless. We define the endowment $w \in L(\mu,\mathbb{R}^2_+)$ by

$$w(a) := \begin{cases} (2,1) & a \in E_1 := [0,.5] \in \Sigma \\ (1,2) & a \in E_2 := (.5,1) \in \Sigma \end{cases}$$

Preferences are given by the following utility function. If $\underline{x} = (x_1,x_2) \in \mathbb{R}^2_+$ then

$$u(a,\underline{x}) := \begin{cases} x_1 & a \in E_1 \\ x_2 & a \in E_2 \end{cases}$$

By a similar reasoning as in example 4.1 we can show that E_x satisfies SCP. Again we establish two major conclusions:

(A) For any $\varepsilon \in (0,.5]$ we define $F_\varepsilon := [0,.5+\varepsilon) \in \Gamma$. A coalition F_ε is able to improve upon w by choosing $f \in L(\mu,\mathbb{R}^2_+)$ defined by

$$f(a) := \begin{cases} (2+2\varepsilon,0) & a \in E_1 \\ (0,2+1/(2\varepsilon)) & a \in E_2 \end{cases}$$

Thus $\int_{F_\varepsilon} f\, d\mu = (1+\varepsilon,2\varepsilon+.5) = \int_{F_\varepsilon} w\, d\mu$. And so $w \notin \tilde{C}(E_x)$.

(B) We claim that there is no contract which is able to improve upon the endowment w. We show this by surveying all possible contracts on realizable coalitions $E \in \Omega(\Gamma)$, with $E = F^* \cup F^\circ$, where $F^* = [0,.5+\varepsilon)$ and $F^\circ = [1-\delta,1)$, for any $0 < \varepsilon+\delta \leq .5$ and $\varepsilon > 0$.

From the definition of the economy E_x we derive that only realizable coalitions of the kind as described above are able to improve upon w. (Other realizable coalitions which are able to improve upon w can be rewritten in this form.)

Now assume $E \in \Omega(\Gamma)$, as described above, is improving upon w with a contract t where $t = c.\chi_{F^*}+d.\chi_{F^\circ}$ $(c,d \in \mathbb{R}^2_+)$. (Other contract on E are of no interest to the problem of blocking.) From the blocking conditions we derive that:

(1) $c_1 > 2$, $c_2 > 2$, and $d_2 > 2$.

(2) $\int_E w\, d\mu \geq (.5+\varepsilon)c+\delta.d$.

Thus $\int_E w_2 \, d\mu = .5+2\varepsilon+2\delta \geq (.5+\varepsilon)c_2+\delta.d_2 > 1+2\varepsilon+2\delta$, which is impossible.

So we conclude that w cannot be blocked by any contract, and therefore we derive that $w \in B(E_x)$.

This example can be interpreted in the same way as is done for example 4.1. However in this example it is shown that there exist economies in which the semi-core is strictly smaller than the contract-core. Thus there exist allocations which can be improved upon by realizable coalitions, but not by any contract. Thus we conclude that some allocations can be blocked only if there is some inequal treatment within primitive coalitions: agents with inequal capacities have to be rewarded differently, also within these primitive coalitions, otherwise these agents are not willing to cooperate in blocking allocations.

□

From these two examples we derive that there exist atomless economies which satisfy the Strong Core Property, such that all core-like equilibrium concepts as introduced in this paper are not equal. From this we may conclude that the conformability property is a necessary condition for the equality of these concepts. Therefore, in connection with theorem 3.9, we conclude that conditions on the coalitional structure such as atomlessness and conformability play a primary rôle in the equivalence results. (As mentioned earlier we will discuss these "structural" conditions more thoroughly in a subsequent paper.)

In Ruys and Gilles (forthcoming) it will be shown that, in line of the work by Aumann and Dreze (1974), Greenberg and Weber (1983) and Kirman, Oddou and Weber (1986), our setting, i.e. modelling with a coalitional structure of agents, can be based on graph-theoretic models. In that paper it also will be questioned if the introduced core-like equilibrium concepts have enough economic content to describe cooperative behavior in these kind of economies fully.

5. CONCLUSIONS.

We summarize the model and results in short:

We introduced a new primitive concept in the modelling of exchange economies with the use of measure theoretic notions: a coalitional structure of agents. An agent is not only an individual, but also has social characteristics and is embedded in an environment. Thus, the agent acts in context of this environment and this leads to the conclusion that the only feasible allocations are those with respect to that environment. Similar it is concluded that the class of feasible coalitions is just the collection of all measurable sets in this structure.

The coalitional structure is build on primitive coalitions, which are the groups of agents which can also act as an entity, i.e. these coalitions are based on a primitive form of interaction or cooperation. From this concept we derived the notions of realizable coalition and (feasible) coalition. In this context we concluded that the "normal" core as defined by Aumann (1964) is adoptable to our setting and that it is based on blocking by <u>all feasible coalitions</u> with positive size. Thus we showed that the "normal" core is based on a quite doubtfull assumption.

Next we introduced two core-like equilibrium concepts: the semi-core and the contract-core. The first one is based on blocking by realizable coalitions only, while the contract-core is based on blocking by contracts, i.e. a very rudimental form of recontracting. Our main results are that under conformability and SCP these new core-like concepts are equal to the normal core and that under atomlessness and conformability the semi-core is equal to the set of Walras allocations.

However we observe two open problems:

(1) Though we achieved a detailed insight into the nature of the conformability condition, as will be displayed in detail in a subsequent paper, we are not able to do the same for the Strong Core Property. Although the SCP seems quite normal, we are not able to state whether many large economies fulfill it and, otherwise, under which simpler conditions it is satisfied. Neither we are able to present an example of a non-trivial economy which does **not** satisfy SCP.

From this short discussion we conclude that in the future we have to search for a full description of the SCP-condition, especially on atomless economies.

(2) The second open problem is that the presented core-like equilibrium concepts are still not fully satisfying: although the semi-core is far more realistic than the "normal" core, because it is based on recontraction by realizable coalitions, it still has the drawback that not all real recontracting forces are described fully. The concept is still too weak. We have to develop new tools for modelling cooperative bahavior in exchange economies, especially recontracting behavior. (This comment can also be given for other approaches, such as the f-core approach.)

Therefore we recommend a further research on the formulation of more realistic core-like equilibrium concepts in settings as presented in this paper. One way of doing this is to construct a setting which is a specification of the one developed in this paper. This is done in the subsequent paper by Ruys and Gilles (forthcoming), in which the authors construct a specific coalitional structure with use of more primitive notions as (economic) capacities of agents and relations between agents. In such settings it is possible to analyse other core-like (or even non-core-like) equilibrium concepts for the cooperative case.

We conclude this section by observing that the model as presented in this paper is just a first step in the direction of more realistic models of exchange economies with core-like equilibrium concepts for the case of cooperative behavior. It may be possible that in such settings the problems as mentioned in Weiss (1981), Armstrong (1985) and Hammond, Kaneko and Wooders (1987) can be solved satisfactory. Secondly it may be possible that this kind of models can be transformed to other areas of economic theory such as non-perfect competitive situations.

APPENDIX

In this appendix we will give the proofs of the main theorems such as presented in section 3. First we state a basic lemma which nearly all proofs are based on.

A.1 Lemma

Let (A,Γ,μ) be a coalitional structure of agents, and $F \in \Sigma$ be a coalition. For every $\varepsilon>0$ there exists a sequence $(E_n)_{n\in\mathbb{N}} \subset \Gamma$ of pairwise disjoint primitive coalitions such that

$$F \subset \bigcup_{n=1}^{\infty} E_n \quad ;$$

$$\sum_{n=1}^{\infty} \mu(E_n) < \mu(F)+\varepsilon \quad .$$

For a proof of this lemma we refer to Janssen and van der Steen (1984) in which it is presented as lemma 3.3.5.

A.2 Proof of theorem 3.3.

Let E_x be an exchange economy and $f \in S(E_x)$.

(a) **Only if**: This is trivial since $\sigma(\Gamma) \subset \Sigma$.
If: First we will show that if $F \in \Sigma$ then there exists a set $E \in \sigma(\Gamma)$ such that $\mu(F) = \mu(E)$ and $E \subset F$.
By lemma A.1 for every $k \in \mathbb{N}$ there exists a set $E_k \in \sigma(\Gamma)$ such that

$$F \subset E_k \text{ and } \mu(E_k) < \mu(F)+2^{-k}.$$

Now put $D:=\bigcap_{k=1}^{\infty} E_k$, then $D \in \sigma(\Gamma)$, $F \subset D$, and $\mu(F) = \mu(D)$. Next consider $B:=D\backslash F$. Since $B \in \Sigma$ and $\mu(B) = 0$, the former statement shows that there exists a set $G \in \sigma(\Gamma)$ with $B \subset G$ and $\mu(G) = 0$. Now choose $E:=D\backslash G$.

Now take f such that it canot be improved upon by any coalition in $\sigma(\Gamma)$. Assume $F \in \Sigma$ is such that $\mu(F) > 0$, and there exists an allocation $g \in L(\mu,\mathbb{R}^{\ell}_{+})$ with $g(a) \succ_a f(a)$, $a \in F$, and $\int_F g \, d\mu \leq \int_F w \, d\mu$.

But then by the assertion above there exists an $E \in \sigma(\Gamma)$ with $E \subset F$ and $\mu(E) = \mu(F) > 0$ and thus E is able to improve upon f by choosing g. This is the required contradiction.

(b) We only have to prove that $\widetilde{B}(E_x) \subset CS(E_x)$, thus: If f can be blocked strongly, then f can also be blocked by a pseudo-contract. The main work is done in the next claim which has an interest on its own:

<u>A.2.1 Claim</u>

Let $\varepsilon > 0$ and $f \in S(E_x)$. If $E \in \Sigma$ is able to improve upon f by choosing $g \in L(\mu,\mathbb{R}^{\ell}_{+})$, then there exists a simple function $\tilde{g}$ on $E \supset G \in \Sigma$ such that

$$0 \leq g(a)-\tilde{g}(a) < \varepsilon.1_{\ell} \qquad , a \in G \in \Sigma \; ;$$

$$\int_{E\setminus G} w \, d\mu < \varepsilon.1_{\ell} \qquad ;$$

$$\int_G \tilde{g} \, d\mu \leq \int_G w \, d\mu \; ,$$

with $1_{\ell}=(1,\ldots,1) \in \mathbb{R}^{\ell}$.

<u>Proof of the claim:</u>

Let $f=(f_1,\ldots,f_{\ell})$, $g=(g_1,\ldots,g_{\ell})$ and $w=(w_1,\ldots,w_{\ell})$ be coordinate-wise representations for f, g, and w.

For every $m \in \{1,\dots,\ell\}$, g_m is a measurable function and according to 2.1.13 in Janssen and van der Steen (1984), every measurable function is the pointwise limit of a sequence of simple functions. Evidently we can take an increasing sequence, and thus we can construct a sequence of increasing simple functions $t_n \uparrow g$ on E.

Now by applying Egorov's theorem to this ℓ-dimensional case we assert that for every $k \in \mathbb{N}$ there exists a set $F_k \subset E$ such that $F_k \in \Sigma$, $\mu(F_k) < 2^{-k}$, and $t_n \uparrow g$ uniformly on $E \backslash F_k$.

It is evident that we can take $F_{k+1} \subset F_k$, $k \in \mathbb{N}$. Thus we can choose $E \supset F \in \Sigma$ such that

$$\int_F w \, d\mu \leq \varepsilon/4.(\mu(G_1),\dots,\mu(G_\ell)) \ ;$$

$$\mu(F) < \min \{ \mu(G_m)/2 \mid 1 \leq m \leq \ell \} \ ;$$

$$t_n \uparrow g \text{ uniformly on } G := E \backslash F,$$

where $G_m := \{ a \in E \mid g_m(a) > \varepsilon \} \in \Sigma$.

[It is clear that we can assume that for all $m=1,\dots,\ell$ $\mu(G_m) > 0$, because otherwise we can take $\tilde{g}_m = 0$ and proceed by deleting the m-th coordinate.]

Now choose $n_\circ \in \mathbb{N}$ such that

$$0 \leq g(a)-t_{n_\circ}(a) < \varepsilon/2.1_\ell, \quad a \in G.$$

(The existence of such a number is garanteed by the uniform convergence of (t_n) on G.)

Now we define:

$$\tilde{g}_m(a) := \begin{cases} t_{n^\circ,m}(a) - \varepsilon/2 & , a \in G \cap G_m. \\ t_{n^\circ,m}(a) & , a \in G \setminus G_m \quad . \end{cases}$$

$\tilde{g} = (\tilde{g}_1, \ldots, \tilde{g}_\ell)$ is a simple function which has the desired properties:

* $\int_G \tilde{g}_m \, d\mu = \int_G t_{n^\circ,m} d\mu - \varepsilon/2.\mu(G \cap G_m) \leq \int_E w_m \, d\mu - \varepsilon/2.\mu(G \cap G_m) \leq \int_G w_m \, d\mu$

because $\mu(G \cap G_m) \geq \mu(G_m) - \mu(F) > \mu(G_m)/2$ and $\int_E w_m d\mu \leq \int_G w_m d\mu + \mu(G_m).\varepsilon/4$.

* If $a \in G \cap G_m$, then

$\tilde{g}_m(a) = t_{n^\circ,m}(a) - \varepsilon/2 > g_m(a) - \varepsilon \ (> 0)$.

If $a \in G \setminus G_m$, then

$0 \leq g_m(a) - \tilde{g}_m(a) = g_m(a) - t_{n^\circ,m}(a) < \varepsilon/2 < \varepsilon$.

Thus $0 \leq g(a) - \tilde{g}(a) < \varepsilon.1_\ell$

* Finally

$$\int_{E \setminus G} w \, d\mu = \int_F w \, d\mu \leq \varepsilon/4.(\mu(G_1), \ldots, \mu(G_\ell)) < \varepsilon.1_\ell ,$$

because $\mu(G_m) \leq 1$, $m=1,\ldots,\ell$.

This concludes the proof of the claim.

Now we proceed the proof of part (b) of theorem 3.3. Let $E \in \Sigma$ with $g \in L(\mu, \mathbb{R}^\ell_+)$ be able to block f in the strong sense. Then by the claim, for every $\varepsilon > 0$, there exists a simple function $\tilde{g}$ such that

$$0 \leq g(a)-\tilde{g}(a) < \varepsilon.1_{\ell} \quad , a \in G \subset E.$$

$$\int_G \tilde{g}\, d\mu \leq \int_G w\, d\mu .$$

Now choose $\varepsilon>0$ small enough such that $\|g-\tilde{g}\|$ is small enough in the sense of the definition 2.4(c). Then $\tilde{g}(a) \succ_a f(a)$, $a \in G$. (Choose $h=\tilde{g}.\chi_G+g.\chi_{E\setminus G}$, then from the definition of strong blocking $h(a) \succ_a f(a)$, $a \in G$.)

Since $\tilde{g}$ is a simple function it forms a pseudo-contract by rewriting $\tilde{g}:=\sum_{n=1}^{N} c_n.\chi_{E_n}$ with $\bigcup_{n=1}^{N} E_n = G$. Thus the theorem is proved.

□

A.3 Proof of theorem 3.7 (a).

Let E_x be a conformable exchange economy. Again we first prove a claim which forms the foundation of the proof, and also has an interest on its own.

A.3.1 Claim.

Let $\varepsilon>0$ and $f \in S(E_x)$. If $E \in \Sigma$ is able to improve upon f by choosing $g \in L(\mu,\mathbb{R}_+^{\ell})$, then there exists a contract t on $E \supset G \in \Omega(\Gamma)$ such that

$$0 \leq g(a)-t(a) < \varepsilon.1_{\ell} \qquad , a \in G \in \Omega(\Gamma) ;$$

$$\int_{E\setminus G} w\, d\mu < \varepsilon.1_{\ell} \qquad ;$$

$$\int_G t\, d\mu \leq \int_G w\, d\mu .$$

Proof of the claim:

By applying claim A.2.1 , there exists a simple function $\tilde{g}$ on

$E \supset H \in \Sigma$ such that

$$0 \leq g(a)-\tilde{g}(a) < \varepsilon/2.1_\ell \qquad (a \in H) \;;$$

$$\int_{E\setminus H} w \, d\mu < \varepsilon/2.1_\ell \qquad ;$$

$$\int_H \tilde{g} \, d\mu \leq \int_H w \, d\mu.$$

Now rewrite $\tilde{g} = \sum_{n=1}^{N} \sigma_n \cdot \chi_{S_n}$ with $S_n \in \Sigma$, pairwise disjoint, $\sigma_n \in \mathbb{R}_+^\ell$ such that $\sigma_n \neq 0$, and $\bigcup_{n=1}^{N} S_n = H$. We construct a contract t as follows:
By conformability, for every $\delta > 0$ and $1 \leq n \leq N$ there exists a coalition $F_n \in \Omega(\Gamma)$, $F_n \subset S_n$ and $\mu(S_n) \leq \mu(F_n)+\delta/(\alpha_n.N)$, where

$\alpha_n := \max\{\sigma_{n,m} \mid 1 \leq m \leq \ell\} > 0 \quad (1 \leq n \leq N)$, and $H\setminus\bigcup_{n=1}^{N} S_n \supset F_{N+1} \in \Omega(\Gamma)$

with $\mu(H\setminus\bigcup_{n=1}^{N} S_n) \leq \mu(F_{N+1})+\delta$.

Now define $\tau := \sum_{n=1}^{N} \sigma_n \cdot \chi_{F_n}$, which is a contract, and $\sigma := \min_n \alpha_n (> 0)$.

Then

$$\int(\tilde{g}-\tau)\, d\mu = \sum_{n=1}^{N} \sigma_n . \mu(S_n \setminus F_n) \leq \sum_{n=1}^{N} \sigma_n . \delta/(\alpha_n . N) \leq \delta . 1_\ell,$$

$$\text{and } \mu(\bigcup_{n=1}^{N+1} S_n \setminus F_n) = \sum_{n=1}^{N+1} \mu(S_n \setminus F_n) \leq \delta/\sigma + \delta.$$

Now we choose $\delta_\circ > 0$ such that

$$\int_{H\setminus \cup F_n} w \, d\mu \leq \varepsilon/4.(\mu(H_1),\dots,\mu(H_\ell)) \text{ and } \mu(F_n) \geq \mu(S_n)/2 \quad (1 \leq n \leq N).$$

where $H_m := \cup\{ S_n \mid \sigma_{n,m} > 0 \}$, $1 \leq m \leq \ell$.

For this $\delta_\circ$ we define $G := \bigcup_{n=1}^{N+1} F_n \in \Omega(\Gamma)$ and

$$t_m(a) := \begin{cases} \tau_m(a) - \varepsilon/2 & a \in G_m. \\ \tau_m(a) & a \notin G_m. \end{cases}$$

with $G_m := \cup\{ F_n \mid \sigma_{n,m} > 0\}$, $1 \leq m \leq \ell$.

Thus t is a non-negative contract, because without loosing generality we can assume that $\sigma_{n,m} \geq \varepsilon/2$ if $F_n \subset G_m$. $(1 \leq m \leq \ell)$. Moreover,

$$\int_G t_m d\mu = \int_{G_m} t_m d\mu = \int_{G_m} \tau_m d\mu - \varepsilon/2.\mu(G_m) = \int_{G_m} \tilde{g}_m d\mu - \varepsilon/2.\mu(G_m) \leq$$

$$\leq \int_H \tilde{g}_m d\mu - \varepsilon/2.\mu(G_m) \leq \int_H w_m d\mu - \varepsilon/2.(G_m) = \int_G w_m d\mu + \int_{H\setminus G} w_m d\mu - \varepsilon/2.\mu(G_m) \leq$$

$$\leq \int_G w_m d\mu + \varepsilon/4.\mu(H_m) - \varepsilon/2.\mu(G_m) \leq \int_G w_m d\mu, \text{ because } \mu(H_m) \leq 2\mu(G_m).$$

$$0 \leq g(a) - t(a) \leq (g(a) - \tilde{g}(a)) + (\tilde{g}(a) - t(a)) < \varepsilon/2.1_\ell + \varepsilon/2.1_\ell = \varepsilon.1_\ell, \ a \in G.$$

$$\int_{E\setminus G} w \, d\mu = \int_{E\setminus H} w \, d\mu + \int_{H\setminus G} w \, d\mu < \varepsilon/2.1_\ell + \varepsilon/4.(\mu(H_1), \ldots, \mu(H_\ell)) \leq$$

$$\leq 3\varepsilon/4.1_\ell < \varepsilon.1_\ell.$$

Thus the contract t has the desired properties, and thus the claim is proved.

Finally we are able to prove theorem 3.7(a), by proving that if $f \in S(E_x)$ can be blocked by a pseudo-contract, then it can be blocked by a contract.

Let E_x have additionally the strong core property. Let $f \in S(E_x)$, which can be blocked by a pseudo-contract. Since E_x satisfies the SCP, there exists (by applying corollary 3.5) a coalition $E \in \Sigma$ which can improve upon f in the strong sense by choosing $g \in L(\mu, \mathbb{R}^\ell_+)$. Now, by the claim, for every $\varepsilon > 0$ there exists a contract t which has the properties as stated in the claim on E and g.

Now choose $\varepsilon > 0$ such that $\| g - t \|$ on $G \subset E$ is small enough that it satisfies the bound given by the definition of strong blocking. Thus we may conclude that $t(a) \succ_a f(a)$, $a \in G$. (Again choose $h = t.\chi_G + g.\chi_{E\setminus G}$, then from the definition of strong blocking we know that $h(a) \succ_a f(a)$, $a \in E$, and thus $t(a) \succ_a f(a)$, $a \in G$.)

Since t is a contract we have proved that any allocation $f \in S(E_x)$, which can be blocked by a pseudo-contract, can also be blocked by a contract.

$\square$

A.4 Proof of theorem 3.9.

In this part we proof that $\tilde{C}(E_x) \subset W(E_x)$.

Let $f \in \tilde{C}(E_x)$ and now we define the multifunction $\varphi: A \twoheadrightarrow \mathbb{R}^{\ell}$ by

$$\varphi(a) := \{ z \in \mathbb{R}^{\ell} \mid z+w(a) \succ_a f(a) \} \cup \{0\}.$$

1) First we proof that φ is a measurable multifunction, i.e. has a measurable graph:

$$\text{Graph}(\varphi) = \{ (a,z) \in A\text{x}\mathbb{R}^{\ell} \mid z+w(a) \succ_a f(a) \} \cup (A \times \{0\}).$$

Since $\{\succ_a\}_{a\in A}$ is measurable and w and f are allocations, i.e. are integrable functions, we see that graph(φ) is a measurable set in $\Sigma \otimes \mathcal{B}(\mathbb{R}^{\ell})$.

2) Since E_x is atomless $\int_A \varphi \, d\mu$ is a convex set. (By application of Lyapunov's theorem.)

3) $\int \varphi \, d\mu \cap \text{int}(\mathbb{R}^{\ell}_-) = \emptyset$.

Assume that there exists $h \in L(\mu,\varphi)$ with $\int h \, d\mu \ll 0$.

Now define

$$S := \{ a \in A \mid h(a) \neq 0 \}.$$

Then $\mu(S) > 0$ and since (A,Γ,μ) is conformable we can take for any $\delta > 0$ a subset $S \supset E \in \Omega(\Gamma)$ with $\mu(S\backslash E) < \delta$.

Now take $\varepsilon > 0$ and choose $\delta > 0$ such that

$$\int_{S\backslash E} \| h \| \, d\mu < \varepsilon, \text{ and } \mu(E) > 0.$$

Then we derive that

$$\int_E h \, d\mu = \int_S h \, d\mu - \int_{S\backslash E} h \, d\mu < \int_S h \, d\mu + \varepsilon.1_\ell.$$

Take $\varepsilon := -\max \{ \int_S h_i \, d\mu \mid i = 1,\dots,\ell \}/2 > 0$ by the property $\int_S h \, d\mu << 0$ and thus

$$\int_E h \, d\mu < \int_S h \, d\mu + \varepsilon.1_\ell << 0 .$$

Now we define $g \in L(\mu,\mathbb{R}^\ell_+)$ as follows:

For $a \in E$: $g(a) := h(a)+w(a)-[\ \mu(E)\ ]^{-1}. \int_E h \, d\mu$.

For $a \notin E$: $g(a) := 0$.

Now E is able to improve upon f by choosing g, since $\mu(E) > 0$ and

$$\int_E g \, d\mu = \int_E h \, d\mu + \int_E w \, d\mu - \int_E h \, d\mu = \int_E w \, d\mu .$$

$$g(a) = h(a) + w(a) - [\ \mu(E)\ ]^{-1}. \int_E h \, d\mu \succ_a h(a) + w(a) \succ_a f(a).$$

(This is true since the preferences are monotone, $\int_E h \, d\mu \ll 0$, and $h \in L(\mu,\varphi)$.)

Finally we are able to follow the lines of the proof of the original Aumann theorem as given by Hildenbrand (1974, p. 134 - 135). All preliminary work is done in the lines above, especially assertions (1), (2) and (3). In his proof Hildenbrand (1974) establishes that there exists a price $p \in \Delta$ such that (f,p) is a Walras equilibrium, and thus it is established that the semi-core of the economy is a subset of the set of all Walras allocations. The existence of such a $p \in \Delta$ is based only on the existence of a separating hyperplane for $\int \varphi \, d\mu$ and $\mathbb{R}^{\ell}_{-}$. Thus the assertion is proved.

□

REFERENCES

Aliprantis, C.D. and O. Burkinshaw (1981), Priciples of real analysis, North-Holland, Amsterdam.

Armstrong, Th.E. (1985), "Remarks related to finitely additive exchange economies.", In: C.D. Aliprantis, O. Burkinshaw and N.J. Rothman (eds.), Advances in equilibrium theory, Springer, Berlin.

Armstrong, Th.E. and M.K. Richter (1984), "The Core-Walras Equivalence.", Journal of mathematical economics, 33, 116 - 151.

Armstrong, Th.E. and M.K. Richter (1986), "Existence of nonatomic Core-Walras allocations.", Journal of economic theory, 38, 137 - 159.

Aumann, R.J. (1964), "Markets with a continuum of traders.", Econometrica, 32, 39 - 50.

Aumann, R.J. (1966), "Existence of competitive equilibria in markets with a continuum of traders.", Econometrica, 34, 1 - 17.

Aumann, R.J. and J. Drèze (1974), "Cooperative games with coalition structures.", International journal of game theory, 3,217 - 237.

de Barra, G. (1981), Measure theory and integration, Wiley, New York.

Bewley, T.F. (1973), "Edgeworth's conjecture.", Econometrica, 41, 425 - 454.

Castaing, C. and M. Valadier (1977), Convex analysis and measurable multi-functions, Springer, Berlin.

Debreu, G. (1982), "Existence of general equilibrium.", In: K.J. Arrow and M.D. Intriligator (eds.), Handbook of Mathematical Economics, North-Holland, Amsterdam.

Greenberg, J. (1986), Stable standards of behavior: A unifying approach to solution concepts., Mimeo.

Greenberg, J. and B. Shitovitz (1986), "A simple proof of the equivalence theorem for oligopolistic mixed markets.", Journal of mathematical economics, 15, 79 - 83.

Greenberg, J. and S. Weber (1983), "A core equivalence theorem with an arbitrary communication structure.", Journal of mathematical economics, 11, 129 - 138.

Grodal, B. (1972), "A second remark on the core of an atomless economy.", Econometrica, 40, 581 - 583.

Halmos, P.R. (1950), Measure theory, Springer, Berlin. (Reprint 1973.)

Hammond, P., M. Kaneko and M.H. Wooders (1987), Continuum economies with finite coalitions: Core, equilibrium, and widespread externalities., Mimeo.

Hildenbrand, W. (1974), Core and Equilibria of a Large Economy, Princeton UP, Princeton.

Hildenbrand, W. (1982), "Core of an economy.", In: K.J. Arrow and M.D. Intriligator (eds.), Handbook of Mathematical Economics, North-Holland, Amsterdam.

Hildenbrand, W. (1986), The Core of an Economy, Discussion paper A-39, Bonn University.

Jacobs, K. (1978), Measure and Integral, Academic Press, New York.

Janssen, A. and P. van der Steen (1984), Integration theory, Springer, Berlin.

Kaneko, M. and M.H. Wooders (1986a), "The core of a game with a continuum of players and finite coalitions: The model and some results.", Mathematical social sciences, 12, 105 - 137.

Kaneko, M. and M.H. Wooders (1986b), The core of a market with a continuum of players and finite coalitions: From finite to continuum economies., Mimeo, IMSSS 94305, Stanford.

Kirman, A. , C. Oddou and S. Weber (1986), "Stochastic communication and coalition formation.", Econometrica, 54, 129 - 138.

Mas-Colell, A. (1985), The theory of general economic equilibrium. A differentiable approach., Cambridge UP, Cambridge.

Oxtoby, J.C. (1980), Measure and Category, Springer, Berlin.

Ruys, P.H.M. and R.P. Gilles (forthcoming), On the Coalitional Structure of a Relational Economy, Tilburg University, Tilburg.

Schmeidler, D. (1972), "A remark on the core of an atomless economy.", Econometrica, 40, 579 - 580.

Shitovitz, B. (1973), "Oligopoly in markets with a continuum of traders.", Econometrica, 41, 467 - 501.

Vind, K. (1972), "A third remark on the core of an atomless economy.", Econometrica, 40, 585 - 586.

Weiss, E-A. (1981), "Finitely additive exchange economies.", Journal of mathematical economies, 8, 221 - 240.

Wooders, M.H. and W. Zame (1984), "Approximate cores of large games.", Econometrica, 52, 1327 - 1350.

Zaanen, A.C. (1967), Integration, North-Holland, Amsterdam.

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